Specialist maths practice 2

1. For what values of a are the vectors (a,a,a), (2,a,a) and (3,4,a) dependent?
2. A and B are defined by the position vectors $\vec{a} = 2\vec{i} - 2\vec{j} - \vec{k}; \vec{b} = 3\vec{i} + 4\vec{k}$
   2.1 Find $\hat{a}, \hat{b}$
   2.2 Find the unit vector which bisects $\angle AOB$
3. A laboratory tank contains 100 litres of a 20% serum solution (i.e. 20% of the contents is pure serum and 80% is distilled water). A 10% serum solution is then pumped in at a rate of 2 l/min, and an amount of the solution currently in the tank is drawn off at the same rate.
   3.1 Set up a differential equation to show the relation between x and t, where x litre is the amount of pure serum in the tank at time t min.
   3.2 How long will it take for there to be an 18% solution in the tank? (Assume that, at all times, the contents of the tank form a uniform solution.)
4. Find the area between the x-axis, $y = \cos^{-1} x, x = \frac{\sqrt{3}}{2}, x = 1$

Ans.

(1) 0,2,4 (2) $\frac{\sqrt{510}}{510}(19\vec{i} - 10\vec{j} + 7\vec{k})$ (3) $\frac{dx}{dt} = \frac{1}{5}\left(1 - \frac{x}{10}\right)$, 11.16 min

(4) $\frac{1}{2} - \frac{\pi\sqrt{3}}{12}$

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