Matrices

A_{23} means the order of Matrix A is "2 by 3". It has 2 rows and 3 columns.

Matrix multiplication :

$A_{23}B_{34} = C_{24}$

The two middle numbers must be the same ("3" in this case). The "order" of the product matrix is the outer numbers (2x4 in this case).

Number algebra	Matrix Algebra
$1*n = n*1 = n$	$IA = AI = A$ for **square matrix** A and the Identity Matrix $I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & : & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ note : the diagonal elements are all "1" in the identity matrix
$n - n = 0$	$A - A = [0]$ the result of the subtraction of the matrix by itself is a "zero matrix", not the number "0". Zero matrix is a matrix in which all the elements are "0". In this case it has the same order as A.
$\frac{n}{n} = 1$	$A^{-1}A = AA^{-1} = I$ • if the inverse exists (note: it may not exists) • there is no division in Matrix operation
$ab = ba$	$\boxed{AB \neq BA}$ in general • **matrix multiplication takes "sides"** : multiply from the left ≠ multiply from the right side in general, even if both matrices are square matrices
$(a+b)c = ac + bc$	$(A+B)C = AC + BC$ right distributive
$c(a+b) = ca + cb$	$C(A+B) = CA + CB$ left distributive
$a(bc) = (ab)c$	$A(BC) = (AB)C$ associative

Multiplication by a row/column matrix

e.g.

$$\begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}=\begin{bmatrix}1 & 2 & 3\end{bmatrix}$$ means "give me the first row of the matrix"

$$\begin{bmatrix}1 & 2 & 0\end{bmatrix}\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}=\begin{bmatrix}9 & 12 & 15\end{bmatrix}$$ means "give me the sum of the first row and 2 times the second row"

$$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}\begin{bmatrix}1\\ 0\\ 0\end{bmatrix}=\begin{bmatrix}1\\ 4\\ 7\end{bmatrix}$$ means "give me the first column of the matrix"

$$\begin{bmatrix}1 & 2 & 3\\ 4 & 5 & 6\\ 7 & 8 & 9\end{bmatrix}\begin{bmatrix}1\\ 2\\ 0\end{bmatrix}=\begin{bmatrix}5\\ 14\\ 23\end{bmatrix}$$ means "give me the sum of the first column and 2 times the second column"

Solving a system of linear equations:

$$Ax = B$$

$$\Rightarrow A^{-1}Ax = A^{-1}B$$

$$\Rightarrow Ix = A^{-1}B$$

$$\Rightarrow x = A^{-1}B$$

e.g.

$$3x+5y-6z=2$$
$$8x+3y-7z=3$$
$$9x-4z=5$$

$$\Rightarrow\begin{bmatrix}3 & 5 & -6\\ 8 & 3 & -7\\ 9 & 0 & -4\end{bmatrix}\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}2\\ 3\\ 5\end{bmatrix}$$

$$\Rightarrow\begin{bmatrix}x\\ y\\ z\end{bmatrix}=\begin{bmatrix}3 & 5 & -6\\ 8 & 3 & -7\\ 9 & 0 & -4\end{bmatrix}^{-1}\begin{bmatrix}2\\ 3\\ 5\end{bmatrix}=\begin{bmatrix}\frac{49}{29}\\ \frac{71}{29}\\ \frac{74}{29}\end{bmatrix}$$

(Note: in some cases there may be no solution, while in other cases, there may be infinite number of solutions.)