Complex numbers

$z = x + iy = rcis(\theta)$

$|z| = \sqrt{x^2 + y^2} = r \in [0, \infty)$,

$|z^n| = |z|^n = r^n$

$Arg(z) = \theta \in (-\pi, \pi]$

$\text{Re}(z) = x$; $\text{Im}(z) = \text{y}$ (NOT iy !)

$x, y \in R$

De Moivre's theorem:

$$(rcis(\theta))^n = r^n cis(n\theta), n \in Z$$

Conversely:

if $z^n = z_1 = rcis(\theta); z_1 \in C$ then

$$z = r^{\frac{1}{n}} cis\left(\frac{\theta + 2k\pi}{n}\right)$$

the arguments of the roots are evenly spaced at $\frac{2\pi}{n}$ around the argand diagram

$z = x + iy; \bar{z} = x - iy$

$z\bar{z} = |z|^2$

$x = \frac{1}{2}(z + \bar{z})$

$y = \frac{1}{2i}(z - \bar{z})$

where $\bar{z}$ is the conjugate of z

$z_1 = r_1 cis(\theta_1); z_2 = r_2 cis(\theta_2)$

$z_1 z_2 = r_1 r_2 cis(\theta_1 + \theta_2 + 2k\pi) \quad k = -1, 0, 1$

$\frac{z_1}{z_2} = \frac{r_1}{r_2} cis(\theta_1 - \theta_2 + 2k\pi) \quad k = -1, 0, 1$

multiplying by i : 90° anticlockwise on the Argand diagram

	z	$rcis(\theta)$	T
i	iz	$rcis\left(\frac{\pi}{2} + \theta\right)$	$\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$
i^2	$-z$	$rcis(\pi + \theta)$	$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$
i^3	$-iz$	$rcis\left(\frac{3\pi}{2} + \theta\right)$	$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$

$\overline{(z_1 - z_2)} = \bar{z}_1 - \bar{z}_2$

$\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$

$\overline{\left(\frac{z_1}{z_2}\right)} = \frac{\bar{z}_1}{\bar{z}_2}$

Nth root of a complex number or a Polynomial

z_1 is the nth root of $z_2 \Rightarrow z_1 = z_2^{\frac{1}{n}}$

	Modulus equal	Equally spaced around Argand Diagram	Complex conjugate roots
Real no.	✓	✓	✓
Complex no.	✓	✓	
P(z) - real coefficient			✓
P(z) - complex coefficient			

Conjugate factor theorem:

If ***all*** a_n in P(z) are **real**, then the complex roots occur in conjugate pairs:
i.e. if $(z - z_1)$ is a factor, so is $(z - \bar{z}_1)$

$-\pi < \arg(z_1 z_2) \le \pi$ because it is the arg of a complex number. $\arg(z_1) + \arg(z_2)$ could be outside this range because it's the addition of 2 numbers.

Fundamental Theorem of Algebra:

$P(z) = a_n z^n + a_{n-1} z^{n-1} + ... + a_1 z + a_0$

$a_n \ne 0, n \in N$

has at least one linear factor in C. It could further deduce that a polynomial of degree n has n roots.

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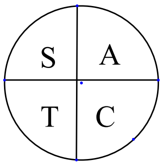
Circular functions



$\sin\left(\frac{\pi}{2}+\theta\right)=\cos\theta$

$\cos\left(\frac{\pi}{2}+\theta\right)=-\sin\theta$

$\sin(a\pm b)=\sin a\cos b\pm\cos a\sin b$

$\cos(a\pm b)=\cos a\cos b\mp\sin a\sin b$

$\tan(a\pm b)=\frac{\tan a\pm\tan b}{1\mp\tan a\tan b}$

$\sin 2x=2\sin x\cos x$

$\cos 2x=2\cos^2 x-1=1-2\sin^2 x$

$\sin^2 x=\frac{1}{2}(1-\cos 2x)$

$\cos^2 x=\frac{1}{2}(1+\cos 2x)$

$\sin a\sin b=\frac{1}{2}[\cos(a-b)-\cos(a+b)]$

$\cos a\cos b=\frac{1}{2}[\cos(a-b)+\cos(a+b)]$

$\sin a\cos b=\frac{1}{2}[\sin(a-b)+\sin(a+b)]$

General Solutions for circular functions:

$\cos(x)=a \quad x=2n\pi\pm\cos^{-1}(a) \quad n\in Z$

$\tan(x)=a \quad x=n\pi+\tan^{-1}(a)$

$\sin(x)=a \quad x=n\pi+(-1)^n\sin^{-1}(a)$

	$\cos^{-1}(x)$	$\sin^{-1}(x)$	$\tan^{-1}(x)$
Dom	$[-1,1]$	$[-1,1]$	R
Ran	$[0,\pi]$	$\left[-\frac{\pi}{2},\frac{\pi}{2}\right]$	$\left(-\frac{\pi}{2},\frac{\pi}{2}\right)$

Even	Odd
cos, sec	sin, cosec, tan, cot, $\sin^{-1}$, $\tan^{-1}$

Sine Rule

$\frac{a}{\sin(A)}=\frac{b}{\sin(B)}=\frac{c}{\sin(C)}$

lookout for ambiguity angle if smaller known side is facing the known angle.

Cosine Rule

$c^2=a^2+b^2-2ab\cos(C)$

Find formula of circular functions from graph

Note: for cot and cosec graphs, $\theta=0$ is the first asymptote and $y>0$ to the immediate right of this asymptote.

e.g. $y=a\,\mathrm{cosec}(bx+c)+d$

1. Find b by determining the period.

   $\text{Period}=\begin{cases}\frac{2\pi}{b} & (\sin,\sec,\cos,\mathrm{cosec})\\ \frac{\pi}{b} & (\tan,\cot)\end{cases}$

2. Find dilation factor a by looking at the y co-ordinates
3. Find translation d by looking at the y co-ordinates
4. Find translation c by comparing the asymptotes or the x co-ordinates. e.g. If the graph for the unknown cosec has an asymptote nearest to the y axis at $x=-\frac{\pi}{6}$ and y > 0 to its right, then

   $bx+c=3\left(-\frac{\pi}{6}\right)+c=\theta=0\Rightarrow c=\frac{\pi}{2}$

	$a\cos(nx)$	$a\sin(nx)$	$a\tan(nx)$
Amplitude	$\lvert a\rvert$	$\lvert a\rvert$	-
Period	$\frac{2\pi}{\lvert n\rvert}$	$\frac{2\pi}{\lvert n\rvert}$	$\frac{\pi}{\lvert n\rvert}$
Range	$[-a,a]$	$[-a,a]$	R
Asymptote	-	-	$\frac{(2k+1)\pi}{2n}, k\in Z$
X intercepts	$\frac{(2k+1)\pi}{2n}, k\in$	$\frac{k\pi}{n}, k\in Z$	$\frac{k\pi}{n}, k\in Z$
x at y_{max}	$\frac{2k\pi}{n}$	$\frac{(1+4k)\pi}{2n}$	-
x at y_{min}	$\frac{(2k+1)\pi}{n}$	$\frac{(3+4k)\pi}{2n}$	-

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General second degree curves

$Ax^2 + Cy^2 + Dx + Ey + F = 0$ (**no cross xy terms!!**)

Shape	Criteria
Line	$A = C = 0$
Circle	$A = C \neq 0$
Ellipse	A, C same sign
hyperbola	A, C opposite sign
Parabola	Either A or C = 0

Differentiation & Integration Table

f(x)	$f'(x)$	$\int f dx$
x^n	nx^{n-1}	$\frac{x^{n+1}}{n+1}$
e^{ax}	ae^{ax}	$\frac{1}{a}e^{ax}$
$\ln\|ax\|$	$\frac{1}{x}$	$x\ln\|ax\| - x$
a^x	$a^x \ln a$	$\frac{a^x}{\ln a}$
$\sin ax$	$a\cos ax$	$\frac{-1}{a}\cos ax$
$\cos ax$	$-a\sin ax$	$\frac{1}{a}\sin ax$
$\tan ax$	$a\sec^2 ax$	$\frac{1}{a}\ln\|\sec ax\|$
$\sin^{-1} ax$	$\frac{a}{\sqrt{1-(ax)^2}}$	Use $\int \boldsymbol{udv} = \boldsymbol{uv} - \int \boldsymbol{vdu}$ Set $\boldsymbol{u} = \sin^{-1} \boldsymbol{ax}$
$\cos^{-1} ax$	$\frac{-a}{\sqrt{1-(ax)^2}}$	Use $\int \boldsymbol{udv} = \boldsymbol{uv} - \int \boldsymbol{vdu}$ Set $\boldsymbol{u} = \cos^{-1} \boldsymbol{ax}$
$\tan^{-1} ax$	$\frac{a}{1+(ax)^2}$	Use $\int \boldsymbol{udv} = \boldsymbol{uv} - \int \boldsymbol{vdu}$ Set $\boldsymbol{u} = \tan^{-1} \boldsymbol{ax}$
$\|u(x)\|$	$\begin{cases} u'(x) & u(x) > 0 \\ -u'(x) & u(x) < 0 \end{cases}$	$\begin{cases} \int udx & u(x) > 0 \\ \int -udx & u(x) < 0 \end{cases}$

	y "		
	<0	0	>0
y ' <> 0	Concave down	Point of inflection	Concave up
y ' = 0	Local max, Concave down	Stationary point of inflection	Local Min, Concave up

Odd and Even functions

Every function can be written as the sum of an odd and an even function:

$$f(x) = \frac{f(x) + f(-x)}{2} + \frac{f(x) - f(-x)}{2}$$

where $g(x) = \frac{f(x) + f(-x)}{2}$ is even,

$h(x) = \frac{f(x) - f(-x)}{2}$ is odd

Functions	Result
Linear combination of Odd	Odd
Linear combination of Even	Even
Linear combination of Even & Odd	Neither even nor odd
Product/Quotient of Odds	Even
Product/Quotients of Evens	Even
Product/Quotients of Odd & Even	Odd
Derivative of Odd	Even
Derivative of Even	Odd
$\int_{-a}^{a} f(x)dx$	$= 0$, $f(x)$ odd $= 2\int_0^a f(x)dx$, $f(x)$ even

Procedure for sketching non-linear graphs

1. Find y and x intercepts
2. Find stationary points ($f'(x) = 0$)
3. Determine nature of stationary points ($f''(x) = ?$ at points in (2))
4. Find points of inflexion ($f''(x) = 0$)
5. Find asymptotes - vertical, horizontal, oblique
6. Label everything - x axis, y-axis, intercepts, graph, turning points, asymptotes

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Applications of Calculus

mixing problems:

$$\frac{dx}{dt} = IR - OR$$

x = amount (mass or volume) of pure substance in container

IR = density/concentration of fluid added (mass or volume of pure substance per unit volume in input) x inflow volume rate

OR = density/concentration of fluid inside (mass or volume of pure substance per unit volume in container) x outflow volume rate

$$\frac{dx}{dt} = C_i \frac{dV_i}{dt} - \frac{x}{V(0) + t\left(\frac{dV_i}{dt} - \frac{dV_o}{dt}\right)} \frac{dV_o}{dt} \quad \text{or}$$

divide both sides by density:

$$\frac{dV_x}{dt} = R_i \frac{dV_i}{dt} - \frac{V_x}{V(0) + t\left(\frac{dV_i}{dt} - \frac{dV_o}{dt}\right)} \frac{dV_o}{dt}$$

x = mass of pure substance in container

V_x = volume of pure substance in container

R_i = ratio of volume of pure substance in input

C_i = input concentration (mass of pure substance per unit volume of input)

$\frac{dV_i}{dt}$ = inflow volume rate

$\frac{dV_o}{dt}$ = outflow volume rate

$V(0)$ = initial volume in container

The solution of one ODE:

$$\frac{dx}{dt} = ax + b \Rightarrow x = ke^{at} - \frac{b}{a}$$

Vectors

$$\underset{\sim}{v} \bullet \underset{\sim}{u} = |\underset{\sim}{v}||\underset{\sim}{u}|\cos\theta$$

$$\underset{\sim}{v} \bullet \underset{\sim}{u} = 0 \Rightarrow \underset{\sim}{v} \perp \underset{\sim}{u} \text{ (perpendicular)}$$

Dependent/independent

A set of vectors $\underset{\sim}{v}_1, \underset{\sim}{v}_2, ...\underset{\sim}{v}_n$ are ***independent*** if the ***only*** linear combination for

$c_1\underset{\sim}{v}_1 + c_2\underset{\sim}{v}_2 + ... + c_n\underset{\sim}{v}_n = \underset{\sim}{0}$; $c_i \in R$

is for all $c_i = 0$. Otherwise, they are dependent.

In matrix language, if we put the vectors into the columns of Matrix A, then the vectors are dependent if Matrix A is rectangular, or its determinant = 0 (Singular matrix).

In a n dimension space, vectors of more than n have at least one dependent vector. (e.g. in 2 dimension, any 3 vectors have at least one dependent vector etc)

A bunch of vectors containing the $\underset{\sim}{0}$ is always dependent

Unit vector :

$$\hat{a} = \frac{\vec{a}}{\|\vec{a}\|}$$

Resolutes :

scalar resolute (a number) = magnitude of v in the direction of u:

$$\underset{\sim}{v} \bullet \underset{\sim}{\hat{u}}$$

vector resolute (a vector) = component of v in the direction of u:

$$\underset{\sim}{v}_{//} = \left(\underset{\sim}{v} \bullet \underset{\sim}{\hat{u}}\right)\underset{\sim}{\hat{u}}$$

component of v $\perp$ to u:

$$\underset{\sim}{v}_{\perp} = \underset{\sim}{v} - \underset{\sim}{v}_{//}$$

vector for angle bisector of $\vec{a}, \vec{b}$:

$\vec{c} = \hat{a} + \hat{b}$; $\hat{a}, \hat{b}$ are **unit vectors**

Angle between $\vec{v} = x\vec{i} + y\vec{j} + z\vec{k}$ and the i, j, k axes:

$$\alpha = \cos^{-1}\left(\frac{x}{|\vec{v}|}\right), \beta = \cos^{-1}\left(\frac{y}{|\vec{v}|}\right), \gamma = \cos^{-1}\left(\frac{z}{|\vec{v}|}\right)$$

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Functions & inverse

Set notation: (e.g)

$\{(x,y): y = x+1, x \in R\}$

$\{(x,y): y = x+1, x \in [1,4)\}$

Mapping notation: (e.g)

$f: R \to R, f(x) = x+1$

$g: [1,4) \to R, g(x) = x+1$

$dom\, f \pm g, fg = dom$ f $\cap\, dom$ g

$f^{-1}[f(x)] = f[f^{-1}(x)] = x$

graph of f^{-1} is a reflection of f in the line y = x

	f	f^{-1}
dom	ran f^{-1}	ran f
ran	dom f^{-1}	dom f

f must be one-to-one for inverse to exist

Composite functions $f \circ g = f(g(x))$

	f	g	$f \circ g$
dom			dom g
ran		ran g $\subset dom$ f	-

If initially ran g $\not\subset$ dom f, we could define g^* such that $ran\ g^* \subset dom$ f by restricting the dom g

Absolute value function

$$|x| = \begin{cases} x & if\ x \ge 0 \\ -x & if\ x < 0 \end{cases}$$

$|x| = \sqrt{x^2}$

$|ab| = |a||b|$

$\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

$|a+b| \le |a| + |b|$ $(= if$ a,b same sign)

Power of x

$f(x) = x^{\frac{p}{q}}$

q even : implied dom R+, y ≥ 0

q odd : function even or odd depend on p even or odd

f(x) is undefined at x = 0 if $\frac{p}{q} < 0$

DRT/RDT transformation

$$\begin{bmatrix} x_1 \\ y_1 \end{bmatrix} = \begin{bmatrix} d_y & 0 \\ 0 & d_x \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}$$

d_x, d_y are dilation(reflection) from x and y axis respectively.

Image $(x_1, y_1) = (d_y x + h, d_x y + k)$

$$\boxed{y_1(x) = d_x y\left(\frac{1}{d_y}(x-h)\right) + k}$$

Dilation factor > 1 graph moves vertically away from x-axis/horizontally away from y-axis (as compared with the basic graph). For dilation factor < 1 graph moves vertically towards x-axis/horizontally towards y-axis (as compared with the basic graph)

TSR/TRD transformation

$$\begin{bmatrix} d_y & 0 \\ 0 & d_x \end{bmatrix}\left(\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Image $(x_1, y_1) = \left(d_y(x+h), d_x(y+k)\right)$

$$y_1(x) = d_x\left[y\left(\frac{1}{d_y}(x - d_y h)\right) + k\right]$$

DTR transformation

$$\begin{bmatrix} r_y & 0 \\ 0 & r_x \end{bmatrix}\left(\begin{bmatrix} d_y & 0 \\ 0 & d_x \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

where $r_y = -1$ if reflection from y axis, otherwise $r_y = 1$, $r_x = -1$ if reflection from x axis, otherwise $r_x = 1$

Image $(x_1, y_1) = \left(r_y(d_y x + h), r_x(d_x y + k)\right)$

$$y_1(x) = r_x\left[d_x y\left(\frac{1}{r_y d_y}(x - r_y h)\right) + k\right]$$

RTD transformation

$$\begin{bmatrix} d_y & 0 \\ 0 & d_x \end{bmatrix}\left(\begin{bmatrix} r_y & 0 \\ 0 & r_x \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} h \\ k \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ y_1 \end{bmatrix}$$

Image $(x_1, y_1) = \left(d_y(r_y x + h), d_x(r_x y + k)\right)$

$$y_1(x) = d_x\left[r_x y\left(\frac{1}{r_y d_y}(x - d_y h)\right) + k\right]$$

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Other transformations

Rotate 90° Anticlockwise about origin

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Rotate 90° clockwise about origin

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

Index, exponential & Log

$a^0 = 1$ $\quad$ $note: a \neq 0$

$a^{\frac{m}{n}} = \sqrt[n]{a^m}$

$a^m \times a^n = a^{m+n}$

$a^m \div a^n = a^{m-n}$

$\left(a^m\right)^n = a^{mn}$

$a^{m^n} = a^{(m^n)}; e.g.\ 2^{2^3} = 2^8 \neq 4^3$

$a^{-m} = \frac{1}{a^m}$

$\log_a(1) = 0$

$\log_a(mn) = \log_a m + \log_a n$

$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n$

$\log_a\left(\frac{1}{m}\right) = -\log_a m$

$\log_a m^n = n \log_a m$

$\log_a x = \frac{\log_b x}{\log_b a}$

$\log_a x$: increasing function for $1 < a$

decreasing function for $0 < a < 1$

let $f(x) = \log_a x \quad \Rightarrow f^{-1}(x) = a^x$

$\log_a a^x = f(f^{-1}(x)) = x$

$a^{\log_a x} = f^{-1}(f(x)) = x$

$a^{-\log_a x} = \frac{1}{x}$

Calculus

$f(x)$ is continuous at $x = a$ if :

1. $f(a)$ is defined
2. $\lim_{x \to a} f(x)$ exists
3. $\lim_{x \to a} f(x) = f(a)$

average rate of change of $f(x)$ between $x \in [x_1, x_2]$

$= \frac{f(x_2) - f(x_1)}{x_2 - x_1}$

instantaneous rate of change of $f(x)$ at $x_1 = f'(x_1)$ where

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

linear approximation (Euler's Formula):

$f(x+h) \approx f(x) + hf'(x)$

This is an overestimate/underestimate if $f(x)$ is concave down/up at x respectively.

Average value of $f(x)$ between x_1 and x_2

$y_{avg} = \frac{1}{(x_2 - x_1)} \int_{x_1}^{x_2} f(x)dx$

Chain Rule

$\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Product Rule

$y = uv \Rightarrow y' = uv' + u'v$

Quotient Rule

$y = \frac{u}{v} \Rightarrow y' = \frac{vu' - uv'}{v^2}$

Note: Derivatives do NOT exist at end points of a domain - need double sided limits. See (2) above.

Equation of tangent to the curve $g(x)$ at $x = x_0$:

Tangent:

$\frac{y - g(x_0)}{x - x_0} = g'(x_0)$

$y = g'(x_0)x - x_0 g'(x_0) + g(x_0)$ (Equation of tangent)

Equation of normal to the curve $g(x)$ $g(x)$ at $x = x_0$:

$\frac{y - g(x_0)}{x - x_0} = \frac{-1}{g'(x_0)}$

$y = \frac{-x}{g'(x_0)} + \frac{x_0}{g'(x_0)} + g(x_0)$ (Equation of normal)

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Some integration methods

Integral	Method
$\sin nx \sin mx$	$\frac{1}{2}\left[\cos(n-m)x-\cos(n+m)x\right]$
$\cos nx \cos mx$	$\frac{1}{2}\left[\cos(n-m)x+\cos(n+m)x\right]$
$\sin nx \cos mx$	$\frac{1}{2}\left[\sin(n-m)x+\sin(n+m)x\right]$
$\sin^n x \cos^m x$	n odd: use $u=\cos x$ or m odd: use $u=\sin x$ n,m both even: use $\sin^2 x=\frac{1}{2}(1-\cos 2x)$ $\cos^2 x=\frac{1}{2}(1+\cos 2x)$
$\frac{(c+dx)^n}{(a+bx)^m}$ n,m +ve integer	u = a + bx, expand numerator using binomial: $(a+x)^n=\sum_{r=0}^{n}\binom{n}{r}a^{n-r}(x)^r$
$\frac{k_1+k_2x}{(ax-bx_1)(cx-dx_2)}$	Use partial fractions: $\frac{A}{(ax-bx_1)}+\frac{B}{(cx-dx_2)}$
$\frac{k_1+k_2x}{(ax-bx_1)^2}$	Use partial fractions: $\frac{A}{(ax-bx_1)}+\frac{B}{(ax-bx_1)^2}$
$\int_a^b vdu$ $\int_a^b v\frac{\partial u}{\partial x}dx$	Integration by parts: $\left[uv\right]_a^b-\int_a^b udv$ $\left[uv\right]_a^b-\int_a^b u\frac{\partial v}{\partial x}dx$
$\int_{-a}^{a} f(x)dx$	$=0$, $f(x)$ odd $=2\int_0^a f(x)dx$, $f(x)$ even

Numerical Method for antidifferentiation for functions that do not readily have antidifferentiations:
e.g.

$$f(b)=\int_a^b \sqrt{\sin(2x)}dx+f(a)$$

The calculator does not know how to calculate the indefinite integral but it knows how to calculate the definite integral. If f(a) is known, f(b) can be found.

Probability & Statistics

$P(\varepsilon)=1$ $\quad \varepsilon = sample$ space

$P(A')=1-P(A)$

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$P(A\cap B)=P(A\mid B)\bullet P(B)$

mutually exclusive : $P(A\cap B)=0$

independent : $P(A\cap B)=P(A)\bullet P(B)$

Binomial Probability:

probability of each trial must be independent of previous trials
probability is constant
each trial has only 2 outcomes – success or failure
the order of outcome is not important

$$P(X=x)=\binom{n}{x}p^x(1-p)^{n-x}$$

$E(x)=np$

$Var(x)=npq$

Discrete probability distributions:

$$E(x)=\mu=\sum_i x_i p(x_i)$$

$$\sigma^2=\sum_i (x_i-\mu)^2 p(x_i)$$

Normal/Standard Normal distribution

$$Z=\frac{x-\mu}{\sigma}$$

Probability within :
$1\sigma=68\%$ $\quad 2\sigma=95\%$ $\quad 3\sigma=99.7\%$

Transition Matrix:

	$X_n=A$	$X_n=B$
$X_{n+1}=A$	$1-a$	b
$X_{n+1}=B$	a	$1-b$

$$\begin{bmatrix}\Pr(X_{n+1}=0)\\ \Pr(X_{n+1}=1)\end{bmatrix}=\begin{bmatrix}1-a & b\\ a & 1-b\end{bmatrix}\begin{bmatrix}\Pr(X_n=0)\\ \Pr(X_n=1)\end{bmatrix}$$

$S_{n+1}=T\cdot S_n$

$S_n=T^n\cdot S_0$

Steady State:

$$\Pr(X_\infty=A)=\frac{b}{a+b}$$

$$\Pr(X_\infty=B)=\frac{a}{a+b}$$

Positively Skewed distribution: Mean > Median
Negatively Skewed distribution: Mean < Median

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Continuous distributions

$$\int_{-\infty}^{\infty} f(x)dx = 1$$

Mean or Expected value E(x) = $\int_{-\infty}^{\infty} xf(x)dx$

$$Var(x) = \sigma^2 = E[(x-\mu)^2] = \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx$$

Median, percentile $q(0.5, 0.75etc) = \int_{-\infty}^{p} f(x)dx$

Mode : $f(M) \geq f(x)$ for all other x

All distributions

$$E(aX+b) = aE(X)+b$$

$$Var(aX+b) = a^2Var(X)$$

$$Var(x) = E\left[(X-\mu)^2\right] = E(X^2) - \left[E(X)\right]^2$$

If X and Y are independent random variables,

$$E(X+Y) = E(X)+E(Y)$$

$$Var(X+Y) = Var(X)+Var(Y)$$

Number of permutations of n distinct objects taken r at a time:

$${}^nP_r = \frac{n!}{(n-r)!}$$

Number of permutations of n objects arranged in a circle: $(n-1)!$

Number of distinct permutations of n things of which n_1 are one kind, n_2 of a second kind,…., n_k of a kth kind :

$$\frac{n!}{n_1!n_2!\cdots n_k!}$$

Number of ways of partitioning a set of n objects into r cells with n_1 elements in the first cell, n_2 elements in the second, and so forth is:

$$\binom{n}{n_1, n_2, \cdots n_r} = \frac{n!}{n_1!n_2!\cdots n_r!}$$

where $n_1 + n_2 + \cdots n_r = n$

Number of combinations of n distinct objects taken r at a time:

$${}^nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$$

Nspire CX CAS calculation

Normcdf(lowbound,upbound,$[\mu,\sigma]$) : calculates the normal ***probability*** between the bounds

invNorm(Area, $[\mu,\sigma]$) :
calculates the ***X*** in the normcdf that corresponds to the area. The lowerbound for the normcdf is $-\infty$

Binomcdf(n,p,lowbound,upbound) :
calculates the binomial ***probability*** between the bounds.
Binompdf(n,p) :
list all the binomial ***probabilities*** in n trials
Binompdf(n,p,x) : calculates the binomial probability of X = x

To find the **sample size** require for a binomial distribution:

1. On a graph page, graph the binomialpdf or binomialcdf, depending whether you need to find the probability for an exact X or for a range of X. Set n = x when entering the distribution function:
   e.g. to find sample size required for a p = 0.3, X = 4:
   f(x) = binompdf (x,0.3,4)
   e.g. to find sample size required for a p = 0.3, X ≥ 3 :
   f(x) = binomcdf(x, 0.3, 3, x)
2. Use Control-T to go to the table page of the graph points
3. scroll down to find the x where the requirement is met. e.g. the probability is ≥ 0.99

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Solid of Revolution
volume of small cyclinder = $\pi r^2 \triangle h$:

revolve around y axis: $V = \pi \int_{y1}^{y2} [x(y)]^2 dy$

revolve around x axis: $V = \pi \int_{x1}^{x2} [y(x)]^2 dx$

Kinematic
Acceleration :

$$a = \frac{dv}{dt} = \frac{dv}{dx} \cdot \frac{dx}{dt} = v\frac{dv}{dx}$$

Tangent vector to a position vector:
Let $\boldsymbol{r(t)} = \boldsymbol{x(t)} \underset{\sim}{\boldsymbol{i}} + \boldsymbol{y(t)} \underset{\sim}{\boldsymbol{j}} + \boldsymbol{z(t)} \underset{\sim}{\boldsymbol{k}}$

The tangent vector to r(t), $\underset{\sim}{\boldsymbol{T}}(\boldsymbol{t}) = \frac{\boldsymbol{dx}}{\boldsymbol{dt}} \underset{\sim}{\boldsymbol{i}} + \frac{\boldsymbol{dy}}{\boldsymbol{dt}} \underset{\sim}{\boldsymbol{j}} + \frac{\boldsymbol{dz}}{\boldsymbol{dt}} \underset{\sim}{\boldsymbol{k}} = \underset{\sim}{\boldsymbol{v}}$

momentum $p = mv$

Dynamic
Lami's Theorem

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Simultaneous Equation

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}^{-1} \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$x = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{\det(A)}; y = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{\det(A)}; z = \frac{\begin{vmatrix} a_{11} & a_{12} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{32} & b_3 \end{vmatrix}}{\det(A)}$$

Area of triangle with vertices
$(x_1, y_1), (x_2, y_2), (x_3, y_3)$:

$$\text{Area} = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

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Quadratic

$$(a+b)^2 = a^2 + b^2 + 2ab$$

$$(a-b)^2 = a^2 + b^2 - 2ab$$

$$a^2 - b^2 = (a+b)(a-b)$$

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$$

Linear graph

$y = mx + b$

y intercept : (0,b)

horizontal line : m = 0

vertical line : m = ∞

$$\frac{y - y_1}{x - x_1} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\frac{x}{a} + \frac{y}{b} = 1$$

x, y intercepts : (a,0),(0,b)

parallel lines : $m_1 = m_2$

perpendicular lines: $m_1 m_2 = -1$

Polynomial

$P(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + ... + a_1 x + a_0$

$a_i \in R,\ n \in Z^+$

$P(x) = 0$:

product of roots:

$$\prod_{i=1}^{n} x_i = x_1 x_2 ... x_n = (-1)^n \frac{a_0}{a_n}$$

sum of roots:

$$\sum_{i=1}^{n} x_i = x_1 + x_2 + ...x_n = -\frac{a_{n-1}}{a_n}$$

Remainder Theorem:

If $P(x)$ is divided by $(ax-b)$, the remainder is $P\left(\frac{b}{a}\right)$

Factor Theorem:

If $(x-a)$ is a factor of $P(x)$, then $P(a)=0$

Quadratic graph

general quadratic graph equation:

$y = ax^2 + bx + c$

$a > 0$: graph concave up, has minimum

$a < 0$: graph concave down, has maximum

can be changed into turning point form:

$$y = a(x - x_{tp})^2 + y_{tp}$$

turning point = $\left(x_{tp}, y_{tp}\right)$

graph is symmetrical at $x = x_{tp}$

$$x_{tp} = \frac{-b}{2a} \qquad y_{tp} = c - ax_{tp}^2$$

Quadratic Equation

$ax^2 + bx + c = 0$

solution: $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Discriminant $D = b^2 - 4ac$

$D > 0$: 2 solutions

$D = 0$: single solution

$D < 0$: no solution

Trigonometry

$a^2 + b^2 = c^2$ (Pythagoras' Theorem)

$$\sin\theta = \frac{O}{H}$$

$$\cos\theta = \frac{A}{H}$$

$$\tan\theta = \frac{O}{A}$$

$$\sin^2\theta + \cos^2\theta = 1$$

special triangles:

$(3,4,5), (1,1,\sqrt{2}), (1,2,\sqrt{3})$

Factorise

$$a_1 x^2 + a_2 x + a_3 = (ax+b)(cx+d)$$

where

$$a_1 = ac$$

$$a_3 = bd$$

$$a_2 = ad + bc$$

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Geometry
Congruent : same shape and size

Congruent triangles: symbol "$\Delta_1 \cong \Delta_2$" if :

AAS : 2 (and hence three) angles equal and a corresponding side

SAS : 2 corresponding sides and their angle in between (included angle). If the angle is not the included angle, the triangles may or may not be congruent

SSS : three corresponding sides equal

RHS: Right angled triangle with hypotenuse and one side (and hence 3 sides) equal

equilateral triangle : 3 equal sides
isosceles triangle : 2 equal sides
scalene triangle : 3 unequal sides
acute-angled triangle : all 3 angles acute
obtuse-angled triangle : one angle obtuse
right-angled triangle
complementary angles : sum of two angles = 90°
supplementary angles : sum of two angles = 180°
(remember : "c" before "s", 90 before 180)
vertically opposite angles are equal
If 2 parallel lines are cut by a third line (the transversal), then : 1. corresponding angles are equal
2. alternate angles are equal
3. co-interior angles are supplementary
sum of interior angles for an n-sided polygon = $(n-2)\cdot 180$
Rhombus is a quadrilateral with all sides equal and parallel opposite sides. A square is a special case of rhombus with all internal angles 90
If sides of similar figures changes by a factor of k, their areas and volumes changes by a factor of k^2 and k^3 respectively
Interquartile range (IQR) = $Q_3 - Q_1$

Scientific notation
Express a number in the form

$$\pm a \text{ x } 10^m \qquad 1 < a < 10, \quad m \in Z$$

e.g. 12.3456 = 1.23456 x 10^1

prefix:
$m = 10^{-3}$ e.g. 1 mm = 1x10^{-3} m
$k = 10^3$ e.g. 1 km = 1x10^3 m

Divisibililty tests

Number	Test
2	Even
3	Digits sum divisible by 3
4	Last 2 digits divisible by 4
5	Ends in 5 or 0
6	Even and divisible by 3
8	Last 3 digits divisible by 8
9	Digits sum divisible by 9

Measurement

Shape	Area	Perimeter
Square	a^2	$4a$
Rectangle	ab	$2(a+b)$
Triangle	$\sqrt{s(s-a)(s-b)(s-c)}$ $\frac{1}{2}(base \cdot height)$ $\frac{1}{2}ab\sin C$	$2s$ $a+b+c$ *note* : $s = \frac{a+b+c}{2}$
Circle	πr^2	$2\pi r$
Trapezium	$\frac{1}{2}(a+b)h$	$a+b+c+d$
parallelogram	bh	$2(a+b)$
sector	$\frac{\theta^\circ}{360^\circ}\pi r^2$	$2r + \frac{\theta}{360}2\pi r$
ellipse	πab	
Cylinder	$2\pi rh + 2\pi r^2$	
Cone	$\pi r^2 + \pi rs$ s = slant side length $s = \sqrt{r^2 + h^2}$	
N sided regular polygon with side x	$\frac{nx^2}{4\tan\left(\frac{180}{n}\right)}$	nx

Shape	Volume
Sphere	$\frac{4}{3}\pi r^3$
Cone	$\frac{1}{3}$(base area x h)
Prism	Base area x height

Unit Conversion
1 km = 1000m
1 m = 100 cm = 1000 mm
1 hr = 3600 s
m/s = (km/hr)/3.6
1 ha (hectare) = 10,000 m^2
1 litre = 1000 cm^3

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Graphs

$$y = ax^m + \frac{b}{x^n} + c$$

has 2 asymptotes :

$x = 0$

$y = ax^m + c$ (m = 0,1)

m = 0 : horizontal asymptote

m = 1 : oblique asymptote

Hyperbola

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

centre (h,k), asymptotes

$$y - k = \pm\frac{b}{a}(x-h)$$

parametric: $\begin{cases} x = h + a\sec t \\ y = k + b\tan t \end{cases}$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

centre (h,k), asymptotes

$$(x-h) = \pm\frac{b}{a}(y-k)$$

Ellipse

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$$

major axis : the larger of a,b

minor axis : the smaller of a,b

parametric: $\begin{cases} x = h + a\cos t \\ y = k + b\sin t \end{cases}$

on complex plane:

$|z-A| + |z-B| = 2a; \quad A, B \in C, a \in R$

circle

$(x-h)^2 + (y-k)^2 = r^2$

parametric: $\begin{cases} x = h + r\cos t \\ y = k + r\sin t \end{cases}$

on complex plane:

$|z - z_1| = r$

Perpendicular Bisector between 2 points z_1, z_2

$|z - z_1| = |z - z_2|$

Fundamental Theorem of Arithmetic

All integers $n > 1$ has a prime power factorisation:

$n = p_1^{e1} p_2^{e2} \ldots p_k^{ek}$ where p_i are distinct prime numbers, ei are positive integers

Test for prime:

1. all primes > 3 are in the form $6n \pm 1$
2. all composite (non-prime) numbers n are divisible by some prime numbers $p < \sqrt{n}$

Further factorisation

$a^n - b^n$ (n odd) can always be factorised :

$a^n - b^n = (a-b)Q(a,b)$ proof: factor theorem

$a^2 + b^2$ can be factorised if $2ab$ is a square number.

e.g.

$a^4 + 4b^4 = (a^2 + 2b^2 + 2ab)(a^2 + 2b^2 - 2ab)$

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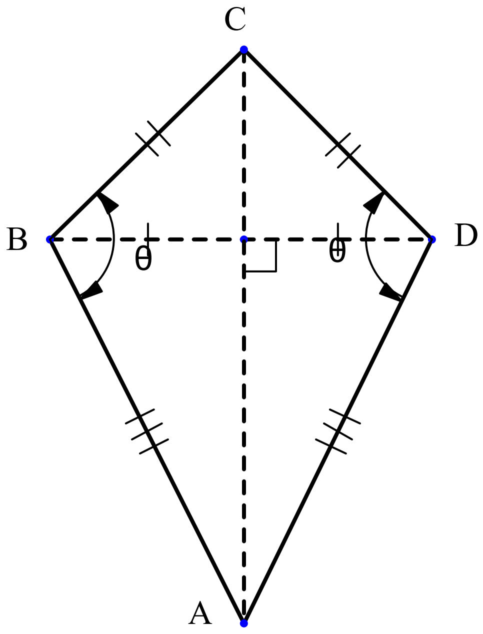
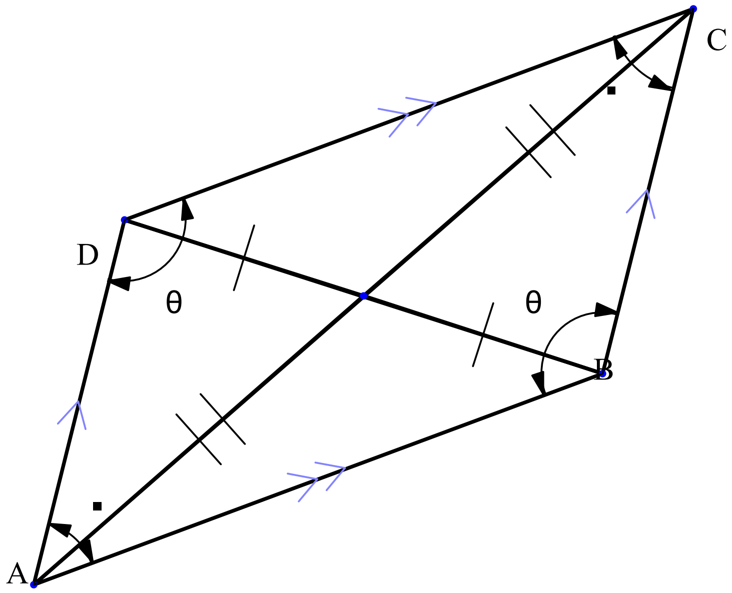
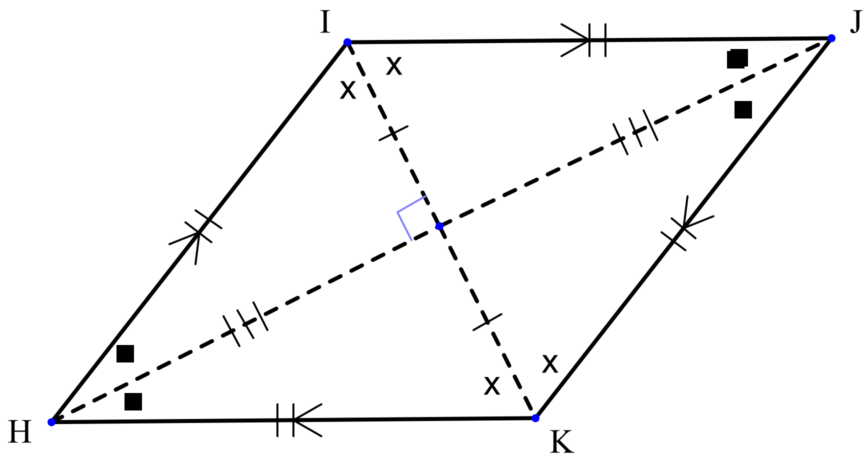


Kite



Parallelogram



Rhombus

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