

Transformation Exercise

1. State the transformation that maps the curve with equation $y = \cos(x)$ to $y = 5 - 6\cos(3x + \pi)$
2. The transformation $T: R^2 \rightarrow R^2$ is defined by $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} 5 & 0 \\ 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 4 \\ -3 \end{bmatrix}$.
The image of the curve $y = 3x^2 + 2$ under the transformation T has equation $y = ax^2 + bx + c$. Find the values of a, b, c.
3. State the transformation that maps $y = e^x$ to $y = 3 - 4e^{-2x+5}$.
4. The transformation $T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \begin{bmatrix} a & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} c \\ d \end{bmatrix}$, $a, c, d \in R$ maps the curve $f(x) = \frac{4}{3x+5}$ to $g(x) = \frac{1}{x}$. Find the values of a, c, d
5. Find the image of the function $y = 4x + 3$ under the transformation $T(X+B)$, where $T = \begin{bmatrix} 0 & 3 \\ -1 & 0 \end{bmatrix}$; $B = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$
6. Find the image of the function $y = \sin(x)$ after the following transformations:
 - translate 3 units in the positive x direction, 1 unit in the negative y direction
 - dilate by a factor of $\frac{1}{2}$ about the x axis
 - reflect about the y axis
7. A transformation is described by $TX+B = X'$ where $T = \begin{bmatrix} -5 & 0 \\ 0 & -3 \end{bmatrix}$ and $B = \begin{bmatrix} 2 \\ -8 \end{bmatrix}$.
Find the image of the curve with equation $y = -3x^3 + 5x^2 + 4$ under the transformation.
8. Describe in words the transformation of the function in (5) above.

Ans

1. dilation from x axis by a factor of 6, reflect from x axis, dilation from y axis by a factor of $\frac{1}{3}$, translate $\frac{\pi}{3}$ in the negative x direction, translate 5 units up

2. $a = \frac{-6}{25}, b = \frac{48}{25}, c = \frac{-271}{25}$

3. dilation from x axis by a factor of 4, reflect from x axis, dilation from y axis by a factor of $\frac{1}{2}$, reflect from y axis, translate $\frac{5}{2}$ units in the positive x direction, translate 3 units up.

4. $a = \frac{-3}{4}, c = \frac{-5}{4}, d = 0$

5. $y_1(x) = -\frac{1}{4}\left(\frac{x}{3} + 3\right)$ 6. $y_1(x) = -\frac{1}{2}(\sin(x+3)+1)$

7. $y_1(x) = \frac{1}{125}(-9x^3 - 21x^2 + 192x - 2728)$

8. Translate 1 unit in the positive x direction, 2 units down. Reflect about the line $y = x$. dilation by a factor of 3 from y axis. reflect about x axis.