Specialist Maths Practice Test 1 (Circular functions, complex numbers, vectors)
Time : 1 hr - Technology Free.

Note : Standard Specialist Maths formulae sheets provided by VCAA can be used.

1. Multiplying a complex number $z(\neq 0)$ by i will

   i) interchange the real and imaginery parts ii) leave the modulus of z unchanged iii) always rotate z counter-clockwise by $\frac{\pi}{2}$ iv) always rotate z clockwise by $\frac{\pi}{2}$

   a) i, ii, iv only b) ii and iv only c) i and iii only d) ii only e) none of the above ans E (2 marks)

2. Let $P(z) = 5z^5 + 6z^4 - 5z^2 + 6 = 0$

   i) P(z) always has 5 roots ii) P(z) has a maximum of 5 roots
   iii) complex roots of P(z) occur in conjugate pairs iv) all roots of P(z) are spaced evenly around the argand diagram v) all roots of P(z) has same modulus

   a) i, iii, iv only b) ii, iv, v only c) i, ii, iii only d) ii, iv, v only
   e) none of the above Ans C (2 marks)

3. The position of a particle at time t is given by $\underset{\sim}{r}(t) = \left(\sqrt{t-3}\right)\underset{\sim}{i} + (3t)\underset{\sim}{j};\quad t \geq 3$. The cartesian equation of the path of the particle is

   a) $y = 3x^2 + 9,\ x \geq 3$ b) $y = 3x^2 + 3,\quad x \geq 3$ c) $y = 3x^2 + 9,\quad x \geq 0$

   d) $y = \sqrt{\frac{x-9}{3}},\quad x \geq 9$ e) $y = 3x^2 + 3,\quad x \geq 0$ ans C (1 marks)

4. In the complex plane, the circle with equation $|z - (2+3i)| = 1$ is intersected exactly twice by the curve with equation

   a) $|z - 3i| = 1$ b) $|z + 3| = |z - 3i|$ c) $|z - 3| = |z - 3i|$ d) $\text{Im}(z) = 4$ e) $\text{Re}(z) = 3$
   ans C (2 marks)

5. For any complex number z, the location on an Argand diagram of the complex number $u = i^3\bar{z}$ can be found by

   a) rotating z through $\frac{3\pi}{2}$ in an anticlockwise direction about the origin

   b) reflecting z about the x-axis and then reflecting about the y-axis

   c) reflecting z about the y-axis and then rotating anticlockwise through $\frac{\pi}{2}$ about the origin

   d) reflecting z about the x-axis and then rotating anticlockwise through $\frac{\pi}{2}$ about the origin

   e) rotating z through $\frac{3\pi}{2}$ in a clockwise direction about the origin

   Ans C (2 marks)

6. Find the general solution for

$$\sqrt{3}\cos x - \sin x = 1$$ (2 marks)

Ans $2n\pi + \frac{\pi}{6}; 2n\pi - \frac{\pi}{2}$

7. Let $\underset{\sim}{u} = \underset{\sim}{i} + 2\underset{\sim}{j} + 3\underset{\sim}{k}; \quad \underset{\sim}{v} = 4\underset{\sim}{i} + 5\underset{\sim}{j} + 6\underset{\sim}{k}; \quad \underset{\sim}{w} = 7\underset{\sim}{i} + 8\underset{\sim}{j} + m\underset{\sim}{k}$. Find the value of m that would make them dependent. Ans 9 (2 marks)

8. Let $\underset{\sim}{u} = 2\underset{\sim}{i} + 3\underset{\sim}{j} + 4\underset{\sim}{k}; \quad \underset{\sim}{v} = 3\underset{\sim}{i} - 5\underset{\sim}{j} + \underset{\sim}{k}; \quad \underset{\sim}{w} = 4\underset{\sim}{i} - 5\underset{\sim}{k}$. Find a vector $\underset{\sim}{p} = a\underset{\sim}{i} + b\underset{\sim}{j} + c\underset{\sim}{k}; \quad a,b,c \in R$ such that $\underset{\sim}{p}$ is perpendicular to $\underset{\sim}{u},\underset{\sim}{v},\underset{\sim}{w}$

Ans $\underset{\sim}{0}$ (2 marks)

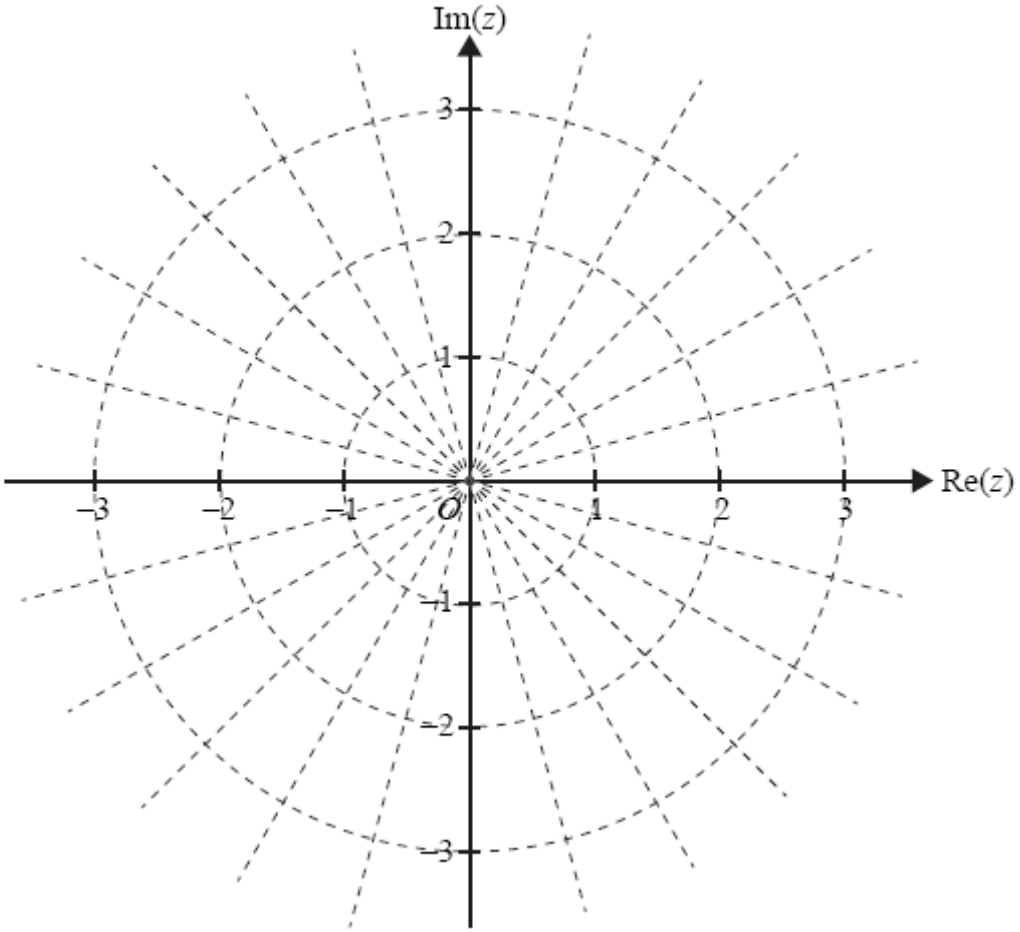
9. Prove that for any 2 vectors $\underset{\sim}{u},\underset{\sim}{v}$ in R^2 ,

$|\underset{\sim}{u} + \underset{\sim}{v}| \le |\underset{\sim}{u}| + |\underset{\sim}{v}|$ if the angle between them is $\le \frac{\pi}{2}$ (3 marks)

10. Given that $z = 2cis\left(\frac{\pi}{6}\right)$. Indicate clearly on the following diagram the position of

$$z, \overline{z}, \frac{1}{\overline{z}}, z - \overline{z}, z\overline{z} - 1$$

(5 marks)



11. Find the exact value of $\sin(75^\circ)$. Given that $\sin(2\theta) = 2\sin\theta\cos\theta$, **hence** find the exact value of $\sin(37.5^\circ)$. Give the reasons for rejecting any solutions.

(2+6 marks)

Ans $\frac{\sqrt{2}(\sqrt{3}+1)}{4}$, $\left(\frac{1-\sqrt{\frac{2-\sqrt{3}}{4}}}{2}\right)^{\frac{1}{2}}$

End of Test

Answer for Spec Practice Test 1

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