Matrices and Determinants Test

1. The order of the matrix $\begin{bmatrix} 2 & 2 \\ 2 & 2 \\ 2 & 2 \end{bmatrix}$ is

   a. 2x2 b. 2x3 c. 3x2 d. 3 e. 6

2. For the matrices $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}; B = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix}$

   i. A = B ii. det(A) = det(B) iii. A is Singular iv. B is Singular
   a. i only b. ii only c. i and ii d. all of the above e. none of the above

3. The total cost of one ice cream and three soft drinks at Catherine's shop is $9. The total cost of two ice creams and five soft drinks is $16. Let x be the cost of an ice cream and y be the cost of a soft drink. The matrix $\begin{bmatrix} x \\ y \end{bmatrix}$ is equal to

   a. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ b. $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$ c. $\begin{bmatrix} 1 & 2 \\ 3 & 5 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$ d. $\begin{bmatrix} -5 & 2 \\ 3 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$ e. $\begin{bmatrix} -5 & 3 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} 9 \\ 16 \end{bmatrix}$

4. A system of three simultaneous linear equations is written in matrix form as follows:

   $$\begin{bmatrix} 1 & -2 & 3 \\ 2 & -4 & 6 \\ 2 & 5 & 8 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 8 \\ -5 \end{bmatrix}$$

   One of the three linear equations is
   a. $x + 2y + 2z = -5$ b. $x - 2y + 3z = -5$ c. $-2x - 4y + 5z = -5$ d. $2x - 4y + 6z = 8$
   e. $2z + 5y + 8x = -5$

5. Let matrix A be m x n and matrix B be r x s, which of the follow is true if the product AB exists (Note: $m \neq n; r \neq s$)

   i. m = r, n = s ii. m = s iii. n = r iv. A, B non-Singular v. order AB is m x s
   a. i only b. ii only c. i,iii,v d. iii, v e. iii,iv, v

6. For square matrices A and B, which of the following properties are true in general:

   i. A + B = B + A ii. AB = BA iii. AB = 0 $\Rightarrow$ A = 0 or B = 0
   iv. $(A+B)^2 = A^2 + B^2 + 2AB$ v. $A^2 - B^2 = (A+B)(A-B)$
   a. i only b. ii only c. i, ii d. i, ii, iii e. all of the above

Questions 7-10 refer to the following matrices:

$A=\begin{bmatrix} a & b \\ c & d \end{bmatrix}; X=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}; E=\begin{bmatrix} e_1 \\ e_2 \end{bmatrix}$ where x_1, x_2 are variables and a, b, c, d, e_1, e_2 are real numbers.

7. The matrix equation AX = E
   i. has a unique solution if A is singular
   ii. has a unique solution if A is non-singular   iii. has no solution if A is singular
   iv. has no solution if A is non-singular
   v. may or may not have infinite solution if A is singular
   a. i only   b. ii only   c. i, iii   d. ii, iv   e. ii, v

8. det(A) =
   i. ad + bc   ii. ad - bc   iii. ab - cd   iv. $\begin{vmatrix} a & b \\ c & d \end{vmatrix}$ v. $\begin{vmatrix} a & c \\ b & d \end{vmatrix}$
   a. i only   b. ii only   c. ii, iv   d. ii, v   e. ii, iv, v

9. $A^{-1} =$
   a. $\frac{-1}{\det(A)}\begin{bmatrix} -d & b \\ c & -a \end{bmatrix}$ b. $\frac{-1}{\det(A)}\begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$ c. $\frac{1}{\det(A)}\begin{bmatrix} -d & b \\ c & -a \end{bmatrix}$
   d. $\frac{1}{\det(A)}\begin{bmatrix} e_1 & x_1 \\ e_2 & x_2 \end{bmatrix}$ e. none of the above

10. Which of the following is true?
    i. $A^{-1} = (EX^{-1})^{-1}$ ii. $X = A^{-1}E$ iii. $X = EA^{-1}$ iv. $E=\begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow X=\begin{bmatrix} 0 \\ 0 \end{bmatrix}$
    v. A is Singular $\Rightarrow \left(EX^{-1}\right)$ is Singular
    a. i only   b. ii only   c. iii only   d. ii, iv, v   e. iii, iv, v

11. $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 5 \\ 6 \end{bmatrix}$. Use Cramer's Rule to find a solution for x, y

12. Prove that $\begin{vmatrix} a+kc & b+kd \\ c & d \end{vmatrix}=\begin{vmatrix} a & b \\ c & d \end{vmatrix}+\begin{vmatrix} kc & kd \\ c & d \end{vmatrix}; k \in R$. If $M=\begin{vmatrix} a+kc & b+kd \\ c & d \end{vmatrix}; N=\begin{vmatrix} a & b \\ c & d \end{vmatrix}$,
    Prove that M = N.

13. $A=\begin{bmatrix} 2r & 6 \\ 7 & 5r \end{bmatrix}$. Find r if A is Singular.

14. $A=\begin{bmatrix} 2r & 5 \\ 4r & 10 \end{bmatrix}$. Find r if A is Singular.

Answers to Matrices and Determinants Test

1c 2b 3e 4d 5d 6a 7e 8e 9a 10b

11 $x=\frac{\begin{vmatrix}5 & 2\\6 & 4\end{vmatrix}}{\begin{vmatrix}1 & 2\\3 & 4\end{vmatrix}}; y=\frac{\begin{vmatrix}1 & 5\\3 & 6\end{vmatrix}}{\begin{vmatrix}1 & 2\\3 & 4\end{vmatrix}}$ 13 $r=\pm\sqrt{\frac{21}{5}}$ 14 $r=R$